

Multiparty Quantum Secret Report

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A multiparty quantum secret report scheme is proposed with quantum encryption. The boss Alice and her M agents first share a sequence of $(M+1)$ -particle Greenberger–Horne–Zeilinger (GHZ) states that only Alice knows which state each $(M+1)$ -particle quantum system is in. Each agent exploits a controlled-not (CNot) gate to encrypt the travelling particle by using the particle in the GHZ state as the control qubit. The boss Alice decrypts the travelling particle with a CNot gate after performing a σ_x operation on her particle in the GHZ state or not. After the GHZ states (the quantum key) are used up, the parties check whether there is a vicious eavesdropper, say Eve, monitoring the quantum line, by picking out some samples from the GHZ states shared and measure them with two measuring bases. After confirming the security of the quantum key, they use the GHZ states remained repeatedly for next round of quantum communication. This scheme has the advantage of high intrinsic efficiency for qubits and the total efficiency.

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The main task of quantum communication is to transmit message securely between the authorized parties and its implement can be completed by the transmission of quantum states.[1, 2] The features of quantum mechanics such as the uncertainty principle, quantum correlations and non-locality play an important role for the security of message.[1] Any of eavesdropper's action will inevitably leave a trace in the result and the parties of communication will detect the eavesdropping by comparing the results of the samples chosen randomly and then abandon all the results transmitted to avoid leaking the secret message.[2] The security of transmission for quantum communication is embodied to the fact that the sender Alice and receiver Bob can find out the malicious or dishonest action of others. Thus quantum communication is secure for generating key [2, 3, 4, 5, 6, 7] and secret message splitting [8, 9, 10, 11, 12, 13, 14, 15, 16] among the parties of communication. Experimental implement for quantum key distribution has been studied by a lot of groups [2]

Recently, the concept of quantum secure direction communication (QSDC), which is used not for distributing key but just for transmitting secret message directly, was proposed and actively pursued. [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31] Shimizu and Imoto [17, 18] and Beige *et al.* [19] proposed some QSDC schemes, in which the secret message can be read out after at least one bit of additional classical information is transmitted for each qubit. In 2002, Boström and Felbinger [27] introduced an insecure ping-pong scheme for

direct communication and key distribution [32]. Subsequently, we [20] put forward a two-step QSDC protocol with Einstein–Podolsky–Rosen (EPR) pairs transmitted in quantum data block and another one based on a sequence of polarized single photons.[21] Wang *et al.*[22] introduced a QSDC protocol with high-dimension quantum superdense coding.

In 2005, Gao *et al.*[33] proposed a scheme for secure direct communication between the central party and the other M parties with Greenberger–Horne–Zeilinger (GHZ) states. In their scheme, the M parties can communicate the central party securely with entanglement swapping. That is, they can send their secret message directly to the central party. We call this model the multiparty quantum secret report (MQSR) as it is used to complete the task that the agents report their secret message to the Boss in one-way direction. In 2006, Jin *et al.* [34] introduced another MQSR scheme with GHZ states following some ideas in quantum dense coding.[35]

In this Letter, we present an MQSR scheme with GHZ states via quantum encryption. The boss Alice and her M agents first share a sequence of $(M+1)$ -particle quantum systems with a decoy technique. The states of the quantum systems are kept by Alice and any other one does not know. Each agent exploits a controlled-not (CNot) gate to encrypt the travelling particle T by using the particle in the GHZ state as the control qubit. The boss Alice decrypts the travelling particle T with a CNot gate after performing a σ_x operation on her particle in the GHZ state or not, according to the fact that her particle is correlated or anti-correlated to that controlled by the agent. After the GHZ states are used up, the parties check whether there is a vicious eavesdropper monitoring the quantum line by selecting some samples

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and measuring them with two measuring bases (MBs), X-MB and Y-MB. After confirming the security of the quantum key, they use the GHZ states remained repeatedly for next round of quantum communication. With quantum storage technique, this scheme has high intrinsic efficiency for qubits and the total efficiency.

Now, let us describe the principle of our MQSR scheme in detail with M agents, say Bob _{r} ($r = 1, 2, \dots, M$).

All the $(M+1)$ -particle GHZ states can be described by [14]

$$\begin{aligned} |G_{\underbrace{j \dots k}_M +}\rangle &= \frac{1}{\sqrt{2}}(|0 \underbrace{j \dots k}_M\rangle + |1 \underbrace{\bar{j} \dots \bar{k}}_M\rangle)_{AB_1 B_2 \dots B_M}, \\ |G_{\underbrace{j \dots k}_M -}\rangle &= \frac{1}{\sqrt{2}}(|0 \underbrace{j \dots k}_M\rangle - |1 \underbrace{\bar{j} \dots \bar{k}}_M\rangle)_{AB_1 B_2 \dots B_M}, \end{aligned} \quad (1)$$

where $j, k \in \{0, 1\}$, \bar{j} and \bar{k} are the counterparts of the binary numbers j and k , respectively; $|0\rangle$ and $|1\rangle$ are the two eigenstates of the Z-MB.

Our MQSR scheme can work with the following steps:

(S1) The Boss Alice shares a sequence of GHZ-state quantum systems, say S_q , with all her agents Bob _{r} ($r = 1, 2, \dots, M$) privately and securely. That is, Alice prepares each quantum system randomly in one of the GHZ states $\{|G_{j \dots k +}\rangle\}$ and keeps the secret information about the states for any one. She sends the particles B_r to the agent Bob _{r} and always controls the particle A for each quantum system. The parties can analyse the security of the GHZ states by the following method. (a) Alice picks out a sufficiently subset of the states as the samples, say s_{e1} for eavesdropping check, similar to Ref. [20]. (b) The agents choose X-MB = $\{|+x\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-x\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$ or Y-MB = $\{|+y\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle), |-y\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)\}$ randomly for their particles in the samples s_{e1} . They tell Alice their MBs for their measurements, and then Alice measures her particles with the correlated MBs. For example, if the state of the GHZ-state quantum system is $|G_{\underbrace{0 \dots 0}_M +}\rangle = \frac{1}{\sqrt{2}}(|0 \underbrace{0 \dots 0}_M\rangle + |1 \underbrace{1 \dots 1}_M\rangle)_{AB_1 B_2 \dots B_M}$, Alice

chooses X-MB for her particle if the number of the agents who choose Y-MB for their measurements on their particles is even, otherwise Alice chooses Y-MB for her measurement. The other cases is the same as this one with or without a little of modification. (c) The agents publish their results of their measurements, and Alice analyses the error rate of the samples. If the error rate is very low, they can conclude that the quantum key, the N ordered GHZ states is created securely. Otherwise, they will discard the results and repeat the quantum communication from the beginning.

For preventing some dishonest agents from stealing the information about the GHZ states with a fake sig-

nal, Alice adds some decoy photons in the sequence S_q . That is, Alice prepares some photons randomly in the states $\{|+x\rangle, |-x\rangle, |+y\rangle, |-y\rangle\}$, with which she replaces some particles B_r in S_q . In this way, the dishonest agent cannot eavesdrop the information for the quantum key S_q with intercepting-resending attack and cheat. Let us use a simple case as an example to describe the principle of this eavesdropping. Assume that there are only two agents, say Bob₁ and Bob₂, and the three-particle quantum system is in the state $|\Psi\rangle_0 \equiv |GHZ\rangle_{00+} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{AB_1 B_2}$. If Bob₁ is a dishonest man, he can intercept the particle B_2 when it runs from Alice to Bob₂. Instead, Bob₁ prepares a Bell state $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{b_1 b_2}$ and sends Bob₂ the particle b_2 and keeps the particle b_1 . When this quantum system is chosen by Alice as a sample for eavesdropping check, Bob₁ performs a Bell-basis measurement on the particles B_2 and b_1 . Then the difference of the state $|\Psi\rangle_0$ and that of the particles $AB_1 b_2$ is just a local unitary operation, I, σ_z, σ_x or $i\sigma_y$,

$$\begin{aligned} |\Psi\rangle_0 &= \frac{1}{2}[(|+x\rangle|+x\rangle + |-x\rangle|-x\rangle)_{B_1 B_2}|+x\rangle_A \\ &\quad + (|+x\rangle|-x\rangle + |-x\rangle|+x\rangle)_{B_1 B_2}|-x\rangle_A] \\ &= \frac{1}{2}[(|+y\rangle|-y\rangle + |-y\rangle|+y\rangle)_{B_1 B_2}|+x\rangle_A \\ &\quad + (|+y\rangle|+y\rangle + |-y\rangle|-y\rangle)_{B_1 B_2}|-x\rangle_A]. \quad (2) \end{aligned}$$

$$\begin{aligned} |\Psi\rangle_0 \otimes |\psi^-\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{AB_1 B_2} \\ &\quad \otimes \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{b_1 b_2} \\ &= \frac{1}{2\sqrt{2}}[(|001\rangle - |110\rangle)_{AB_1 b_2}|\phi^+\rangle_{B_2 b_1} \\ &\quad + (|001\rangle + |110\rangle)_{AB_1 b_2}|\phi^-\rangle_{B_2 b_1} \\ &\quad - (|000\rangle - |111\rangle)_{AB_1 b_2}|\psi^+\rangle_{B_2 b_1} \\ &\quad - (|000\rangle + |111\rangle)_{AB_1 b_2}|\psi^-\rangle_{B_2 b_1}], \quad (3) \end{aligned}$$

where

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad |\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle). \quad (4)$$

Bob₁ can hide the difference with cheat. Assume that the final state of the particles $AB_1 b_2$ is

$$\begin{aligned} |\Psi\rangle_1 &\equiv |GHZ\rangle_{00-} = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \\ &= \frac{1}{2}[(|+x\rangle|-x\rangle + |-x\rangle|+x\rangle)_{B_1 b_2}|+x\rangle_A \\ &\quad + (|+x\rangle|+x\rangle + |-x\rangle|-x\rangle)_{B_1 b_2}|-x\rangle_A] \\ &= \frac{1}{2}[(|+y\rangle|+y\rangle + |-y\rangle|-y\rangle)_{B_1 b_2}|+x\rangle_A \\ &\quad + (|+y\rangle|-y\rangle + |-y\rangle|+y\rangle)_{B_1 b_2}|-x\rangle_A]. \quad (5) \end{aligned}$$

Bob₁ can publish a counterpart of the result of his measurement for his attack when the quantum system

AB_1B_2 is chosen for eavesdropping check, i.e., if he measures his particle B_1 and obtains his result $|+x\rangle$, he tells Alice that his result is $|+x\rangle$. Bob₁'s cheat cannot be detected. The cases with the final states $\frac{1}{\sqrt{2}}(|001\rangle \pm |110\rangle)_{AB_1B_2}$ are the same as this one with or without a little of modification. Fortunately, this attack does not work for the decoy photons as its security is the same as the Bennett-Brassard quantum key distribution protocol. [3, 36] Thus the parties can share a sequence of GHZ states securely.

(S2) The agents use the GHZ states as the quantum key and send their secret message to the boss Alice with a controlled-not (CNot) gate, similar to Ref. [37] (the difference is just that the quantum key is unknown for the agents in this scheme). In detail, an agent, say Bob_r, can report his secret message to the boss Alice by means that he prepares a travelling particle T in the state $|\alpha\rangle_T \in \{|0\rangle, |1\rangle\}$, which represents the classical bit 0 and 1 and takes a CNot operation on the particle T by using the qubit B_r as the control qubit. For instance, we assume that the GHZ state in the quantum key is

$$|G\rangle = \frac{1}{\sqrt{2}}(|0j \cdots i \cdots k\rangle + |\bar{1}\bar{j} \cdots \bar{i} \cdots \bar{k}\rangle)_{AB_1B_2 \cdots B_r \cdots B_M}. \quad (6)$$

After the encryption carried out by Bob_r with a CNot operation, the state $|\Psi\rangle_{s1}$ of the whole quantum system composed of the GHZ-state particles and the travelling particle T is

$$|\Psi\rangle_{s1} = \frac{1}{\sqrt{2}}(|0j \cdots i \cdots k\rangle|i \oplus \alpha\rangle + |\bar{1}\bar{j} \cdots \bar{i} \cdots \bar{k}\rangle|\bar{i} \oplus \alpha\rangle)_{AB_1B_2 \cdots B_r \cdots B_M T}. \quad (7)$$

Bob_r sends the particle T to Alice who can decrypt the qubit T with a CNot operation on the particles T and A by using her qubit A as the control qubit after flipping its bit value or not (if the qubits B_r and A is anti-correlated, Alice should first operate the qubit A with the Pauli operation σ_x). The agent Bob_r continues the quantum communication until the quantum key is used up.

In order to estimate the error rate of the transmission for the T sequence which is composed of all the travelling particles sent by Bob_r, he should insert some sample particles s_{e3} in the T sequence before it runs in the quantum line. The number of the particles in s_{e3} is not required to be very large, but enough for the statistical analysis. As any eavesdropper does not know the information about the GHZ states, he cannot decrypt the encryption done by Bob_r. Thus the security of the quantum communication between Alice and Bob_r is the same as that in quantum one-time pad.[2]

(S3) The agents and Alice use repeatedly the quantum key conditionally, similar to Ref. [38]. That is, they first check the security of the quantum key by choosing a subset of the GHZ states and measuring them with the two MBs, X-MB and Y-MB, same as that in the process

for creating the quantum key. If Alice confirms that the quantum key is secure, the parties use the GHZ states remained again. As the amount of the samples chosen for checking eavesdropping is negligible comparing with all the GHZ states, the quantum key is used repeatedly in the next round if there is no one monitoring the quantum line.

For saving the GHZ states in the quantum key in the process for eavesdropping check, Alice and Bob_r can first choose the GHZ states used for encrypting the sample particles s_{e3} as the samples for checking the security of the quantum key. Then they choose some other GHZ states in the sequence S_q for the eavesdropping check.

(S4) The agents and Alice repeat the steps (2) and (3) until their task is accomplished.

Different from the classical key in one-time pad cryptosystem, the quantum key in this MQSR scheme can be repeatedly used except for the states chosen for checking eavesdropping. For the eavesdropper Eve the quantum key is randomly in one of the two eigenstates of her measuring operation, and then she cannot decrypt the qubits in the quantum key. Moreover, her action will destroy the correlation of the particles in a GHZ state and will be detected by the authorized parties. The randomness of the quantum key for the eavesdropper ensures the security of the secret message transmitted.

With quantum storage technique,[39] this MQSR scheme has the advantage of high intrinsic efficiency for qubits $\eta_q \equiv \frac{q_u}{q_t}$ since almost all qubits are useful in principle for transmitting the secret message. Here q_u and q_t are the qubits useful and total qubits for the transmission, respectively. The total efficiency η_t in this scheme approaches 100% as the classical information exchanged is unnecessary except for the eavesdropping check. Here η is defined as [40]

$$\eta_t = \frac{q_u}{q_t + b_t}, \quad (8)$$

where b_t is the classical bits exchanged between the parties in the quantum communication.

In summary, we have presented a scheme for multi-party quantum secret report with quantum encryption. In this scheme, the Boss Alice first shares a sequence of GHZ states as the quantum key, and then the agents use a CNot operation to encrypt their message and then send it to the Boss. The states of the GHZ quantum systems are unknown for any of the agents, and the quantum communication is secure same as a quantum one-time pad cryptosystem. They can use the quantum key repeatedly except for the GHZ states used for check eavesdropping if there is no-one monitoring the quantum line. This scheme has advantage of having high intrinsic efficiency for qubits η_q and the total efficiency η_t as almost all the qubits are useful for carrying the secret message and the classical information exchanged is unnecessary for the qubits but those chosen for check eavesdropping.

On the other hand, the parties should have the capability of storing the GHZ states.

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